15.2

The Kibble balance compares electrical and mechanical power:

$$IV = mgv$$

Measure V with the Josephson effect:

$$V = n_2 \frac{h}{2e} f_2$$

Measure I indirectly:

$$V_R = IR$$

Measure V_R with the Josephson effect:

$$V_R = n_1 \frac{h}{2e} f_1$$

If $R = R_H = \frac{h}{ie^2}$

$$I = \frac{V_R}{R} = \frac{n_1 \frac{h}{2e} f_1}{\frac{h}{ie^2}} = n_1 f_1 \frac{h}{2e} \cdot \frac{ie^2}{h} = n_1 f_1 \frac{ie}{2}$$

$$IV = \left(n_1 f_1 \frac{ie}{2}\right)\left(n_2 \frac{h}{2e} f_2\right) = mgv$$

$$n_1 n_2 f_1 f_2 \frac{ie}{2} \cdot \frac{h}{2e} = mgv$$

$$n_1 n_2 f_1 f_2 \frac{ih}{4} = mgv$$

$$m = \frac{n_1 n_2 f_1 f_2 \frac{ih}{4}}{gv}$$

$\boxed{15.3}$ From page 254:

$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}\ Wb\ (\text{SI units})$$

Flux $\Phi = BA$, so minimum detectable field:

$$B_{min} = \frac{\Phi_0}{A}$$

$$A = 1 cm^2 = 10^{-4} m^2$$

$$B_{min} = \frac{2.07 \times 10^{-15}}{10^{-4}} = 2.07 \times 10^{-11}\ T$$

For a long wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 = 4\pi \times 10^{-7} H/m.\ I = 1A$$

$$B = \frac{4\pi \times 10^{-7} \times 1}{2\pi r} = \frac{2 \times 10^{-7}}{r}$$

Solve for r:

$$\frac{2 \times 10^{-7}}{r} = 2.07 \times 10^{-11}$$

$$r = \frac{2 \times 10^{-7}}{2.07 \times 10^{-11}} \simeq 9660 m = 9.66 km$$

$\boxed{15.5}$

Time Error

1 month = $30 \times 86400 = 2.592 \times 10^6$ s

Harrison = $\Delta t = 10^{-5} \times 2.592 \times 10^6 \approx 25.92$ s.

Cesium : $\Delta t = 10^{-12} \times 2.592 \times 10^6 \approx 2.592 \times 10^{-6}$ s

Position Error

1s time error = 15" longitude = 463 m at equator

Harrison : $463 \times 25.92 \approx 12{,}000$ m = 12 km

Cesium : $463 \times 2.592 \times 10^{-6} \approx 0.0012$ m = 1.2 mm

15.6

Given: Altitude = 20180 km, $r = 6371 + 20180 = 26551$ km

a) Speed:

$$v = \sqrt{\frac{GM}{r}}, \quad GM = 3.986 \times 10^{14}\ m^3/s^2$$

$$v = \sqrt{\frac{3.986 \times 10^{14}}{2.6551 \times 10^{7}}} \simeq 3875.6\ m/s$$

b) Period:

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 2.6551 \times 10^{7}}{3875.6} \simeq 4.299 \times 10^{4}\ s \Rightarrow 11.94\ hours$$

c) Speed Relativity:

$$\Delta t = \frac{1}{2} T \left(\frac{v}{c}\right)^2, \quad \frac{v}{c} = \frac{3875.6}{3 \times 10^{8}} \simeq 1.292 \times 10^{-5}$$

$$\Delta t \simeq \frac{1}{2} \times 4.299 \times 10^{4} \times 1.67 \times 10^{-10} \simeq 3.59\ \mu s$$

Satellite clock slower.

d) General Relativity:

$$\Delta t = T\left(\frac{GM}{R_E c^2} - \frac{GM}{r c^2}\right)$$

$$\frac{GM}{c^2} = 4.429 \times 10^{-3}\ m$$

$$\Delta t \simeq 4.299 \times 10^{4} \times 5.28 \times 10^{-10} \simeq 22.7\ \mu s$$